

Analysis And Measurement Of Intrinsic Noise In Op Amp Circuits Part II: Introduction To Op Amp Noise

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An important characteristic of noise is its spectral density. Voltage noise spectral density is a measurement of root-mean-square (rms) noise voltage per square root Hertz (or commonly: nV/ $\sqrt{\text{Hz}}$). Power spectral density is given in W/Hz. In the previous article we learned that the thermal noise of a resistor can be computed using Equation 2.1. This equation can be rearranged into a spectral density form. One important characteristic of this noise is that it has a flat spectral density plot (ie it has uniform energy at all frequencies). For this reason, thermal noise is sometimes called broadband noise. Op amps also have broadband noise associated with them. Broadband noise is defined as noise that has a flat spectral density plot.

$$e_n = \sqrt{4kTR\Delta f} \quad \text{Format used in Part 1}$$

$$\frac{e_n}{\sqrt{\text{Hz}}} = \sqrt{4kTR} \quad \text{Spectral Density Format}$$

Eq 2.1

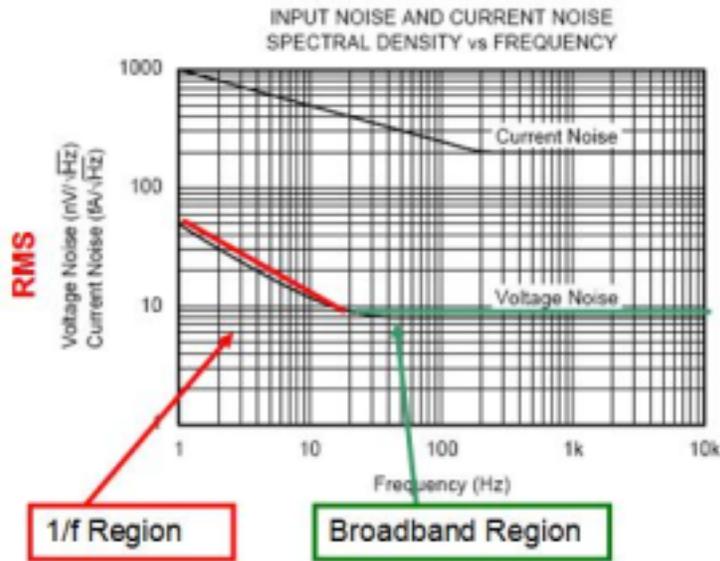


Fig. 2.1: Op Amp Noise Spectral Density

In addition to broadband noise, op amps often have a low-frequency noise region that does not have a flat spectral density plot. This noise is called 1/f noise, flicker noise, or low-frequency noise. Typically, the power spectrum of 1/f noise falls at a rate of 1/f. This means that the voltage spectrum falls at a rate of 1/f^(1/2). In practice, however, the exponent of the 1/f function may deviate slightly. Fig. 2.1 shows a typical op amp spectrum with both a 1/f region and a broadband region. Note that the spectral density plot also shows current noise (given in fA/ $\sqrt{\text{Hz}}$).

It is important to note that $1/f$ noise also has a normal distribution and, consequently, the mathematics described in Part I still apply. Fig. 2.2 shows the time domain description of $1/f$ noise. Notice that the x-axis of this graph is given in seconds; this slow change with time is typical for $1/f$ noise.

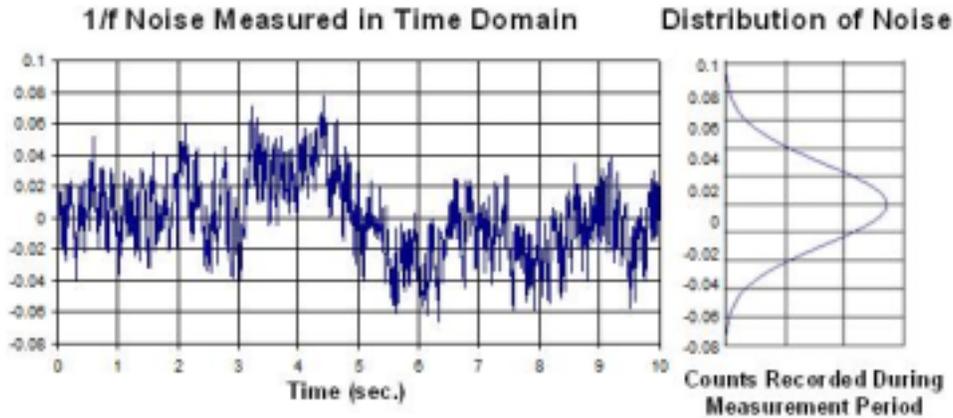


Fig. 2.2: $1/f$ Noise Shown In The Time Domain And Statistically

The standard model for op amp noise is shown in Fig. 2.3. It consists of two uncorrelated current noise sources and one voltage noise source connected to the op amp inputs. The voltage noise source can be thought of as time-varying input offset voltage component, and the current noise sources can be thought of time-varying bias current components.

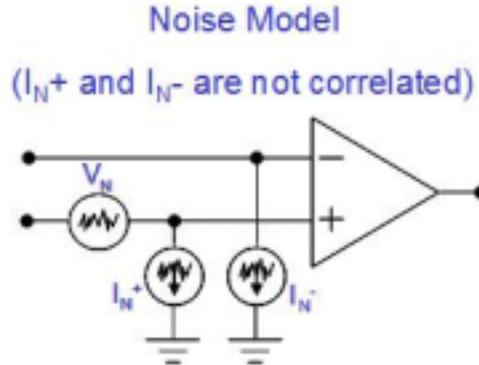


Fig. 2.3: Op Amp Noise Model

Op Amp Noise Analysis Technique

The goal of op amp noise analysis technique is to calculate the peak-to-peak output noise based on op amp data sheet information. As the technique is explained, we will use formulas that apply to most simple op amp circuits. For more complex circuits, the formulas can help to get a rough idea of the expected noise output. It is possible to develop more accurate formulas for these complex circuits; however, the math would be overly complex. For the complex circuits, it is probably best to use a three-step approach. First get a rough estimate using the formulas, second get a more accurate estimate using spice, and finally verify your results through measurements.

As an example circuit, we will use a simple non-inverting amplifier with a TI OPA277 (see Fig. 2.4). Our goal is to determine the peak-to-peak output noise and to do this we have to consider the op amp's current noise, voltage noise, and the resistor thermal noise. We will determine the value of these noise sources using the spectral density curves in the data sheet. Also, we will have to consider the gain and bandwidth of the circuit.

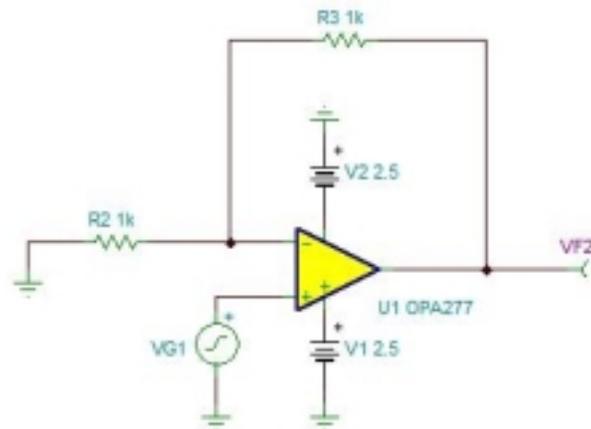


Fig. 2.4: Example Circuit For Noise Analysis

First, we must understand how to convert the noise spectral density curves to a noise source. In order to do this we will have to use some calculus. As a quick reminder, the integral function will give the area under a curve. Fig. 2.5 shows how a constant function can be integrated by simply multiplying the height times the width (ie the area of a rectangle). This simple relationship converts the spectral density curves to noise sources.

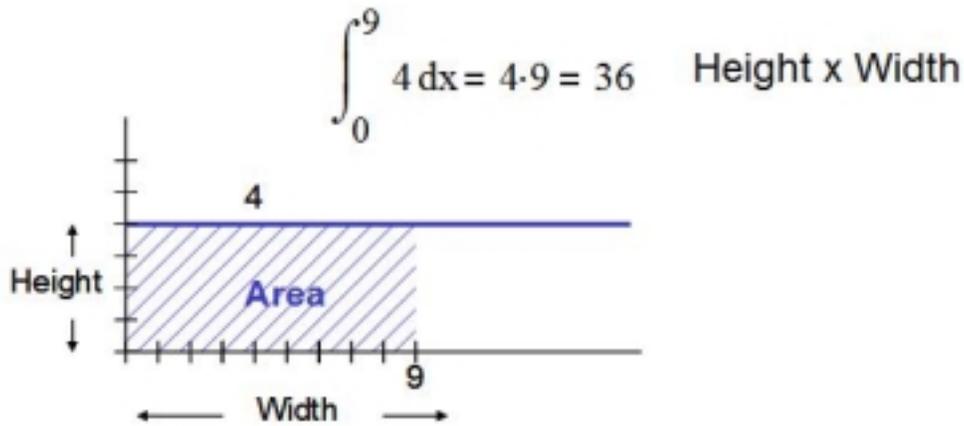


Fig. 2.5: Integration Computes Area Under A Curve

People will often say that you must integrate the voltage spectral density curve to get total noise. In reality, you must integrate the power spectral density curve. This curve is simply the voltage or current spectral density squared (remember $P = V^2/R$ and $P = I^2R$). Fig. 2.6 shows the strange units that result when you attempt to integrate the voltage spectral density curve. Fig. 2.7 shows how you can integrate the power spectral density and convert back to voltage by taking the square root of the result. Note that we get the proper units.

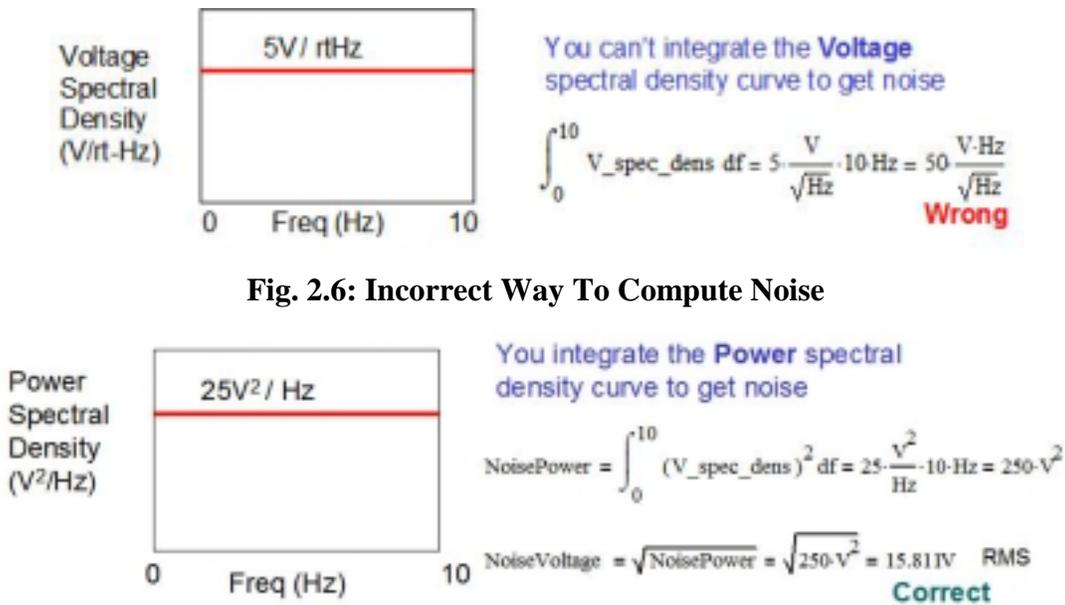


Fig. 2.7: Correct Way To Compute Noise

Integrating the power spectral density curve for the voltage and current spectrums will give us the rms magnitude of the sources in the op amp model (Fig. 2.3). However, the shape of the spectral density curve will contain a 1/f region and a broad band region with a low-pass filter (see Fig. 2.8). Calculating the total noise of these two sections will

require the use of formulas that were derived using calculus. The results of these two computations are added using root-sum square (rss) addition for uncorrelated sources that was discussed in Part I.

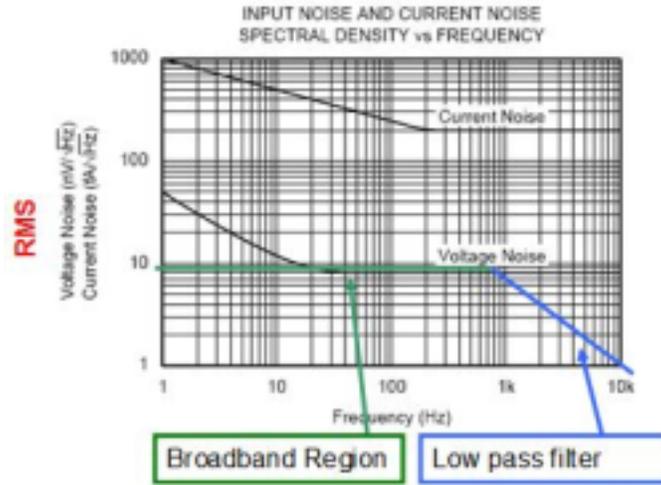


Fig. 2.8: Broadband Region With Filter

First, we will integrate the broadband region with a low-pass filter. Ideally, the low-pass filter portion of this curve would be a straight vertical line. This is referred to as a brick-wall filter. Solving the area under a brick-wall filter is easy because it is a rectangle (height \times width). In the real world we cannot realize a brick-wall filter. However, there are a set of constants that can be used to convert real-world filter bandwidth to an equivalent brick-wall filter bandwidth for the purpose of the noise calculation. Fig. 2.9 compares the theoretical brick-wall filter to first-, second- and third-order filters.

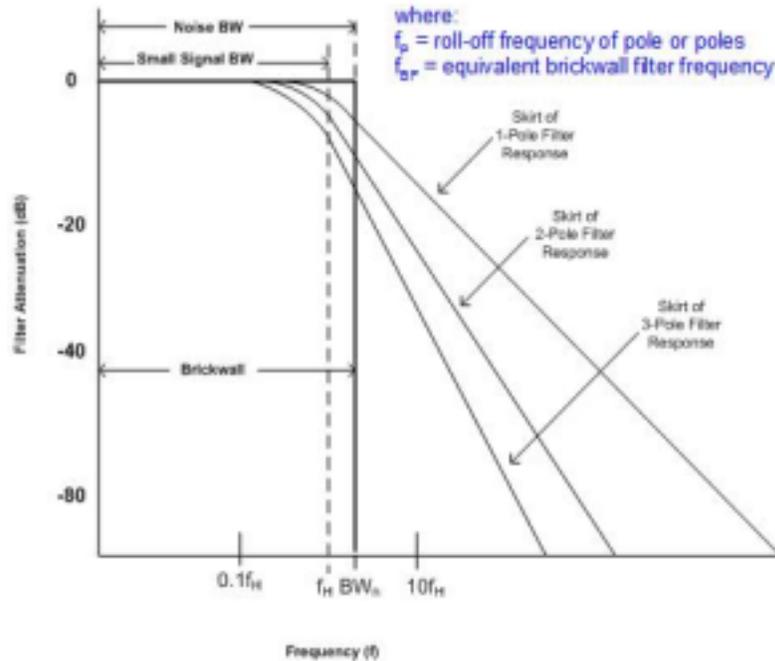


Fig. 2.9: Comparison Of Brick-Wall Filter To Real-World Filter

The next equation is used to convert the real-world filter or the brick-wall equivalent. Table 2.1 lists the brick-wall conversion factors (K_n) for different filter orders. For example, a first-order filter bandwidth can be converted to a brick-wall filter bandwidth by multiplying by 1.57. The adjusted bandwidth is sometimes referred to as the noise bandwidth. Note that the conversion factor approaches one as the order increases. In other words, higher-order filters are a better approximation of a brick-wall filter.

$$BW_n = f_H \cdot K_n \quad \text{Effective Noise Bandwidth}$$

where

f_H - is the upper cut frequency

K_n - is the brick wall conversion factor

Eq 2.2

Number of Poles in Filter	K_n Ac Noise Bandwidth Ratio
1	1.57
2	1.22
3	1.16
4	1.13
5	1.12

Table 2.1: Brick-Wall Correction Factor

So now that we have a formula to convert a real-world filter to its brick-wall equivalent, it is a simple matter to integrate the power spectrum. Remember, integrating the power is the voltage spectrum squared. At the end of the integration, the square root is taken to convert back to voltage. The next equation was derived in this manor (see Appendix 2.1). This, and the last equation, are used in conjunction with the data sheet information to determine the broadband noise contribution.

$$e_{nBB} = e_{BB} \sqrt{BW_n}$$

where

e_{nBB} - Broadband voltage noise in volts rms

e_{BB} - Broadband voltage noise density; usually in nV/rt-Hz

BW_n - Noise bandwidth for a given system

Eq 2.3

Recall that our goal is to determine the magnitude of the noise source V_n from Fig. 2.3. This noise source consists of both broadband noise and $1/f$ noise. Using the last two equations we were able to compute the broadband component. Now we need to compute the $1/f$ component. This is done by integrating the power spectrum of the $1/f$ region of the noise spectral density plot. Fig. 2.10 shows this region graphically.

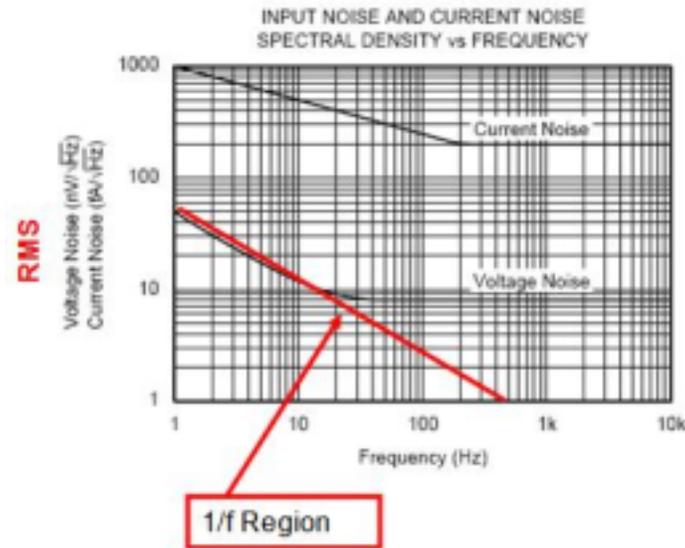


Fig. 2.10: 1/f Region

The result of the integration is given by the two equations following, the first normalizing any noise measurement in the $1/f$ region to the noise at 1 Hz. In some cases this number can be read directly from the chart, in other cases it is more convenient to use this equation (see Fig. 2.11). The second computes the $1/f$ noise using the normalized noise, upper noise bandwidth, and lower noise bandwidth. The full derivation is given in Appendix 2.2.

$$e_{f\text{norm}} = e_{\text{at}_f} \sqrt{f}$$

Where

$e_{f\text{norm}}$ - normalized noise at 1Hz (usually in nV)

e_{at_f} - voltage noise density at f (usually in nV/rt-Hz)

f - a frequency in the $1/f$ region where noise voltage density is known

Eq 2.4

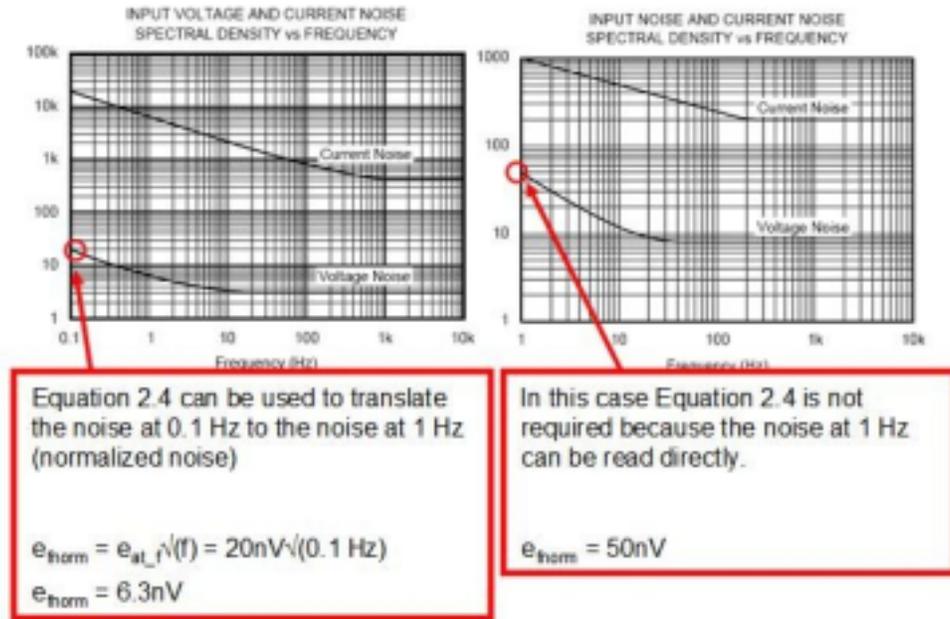


Fig. 2.11: Two 1/f Normalizing Cases

$$e_{\text{nf}} = e_{\text{norm}} \cdot \sqrt{\ln\left(\frac{f_H}{f_L}\right)}$$

Where

e_{nf} - 1/f noise in volts rms

e_{norm} - normalized noise at 1Hz from Equation 4 (usually in nV)

f_H - upper frequency of operation (use the noise bandwidth BW_n)

f_L - lower cut frequency of operation (0.1Hz is typically used).

Eq 2.5

When considering the 1/f noise you must choose a low-frequency cutoff. This is because the 1/f function is not defined at zero (ie 1/0 is undefined). In fact, the noise theoretically goes to infinity when you integrate back to zero Hertz. However, you should consider that very low frequencies correspond to long times. For example, 0.1 Hz corresponds to 10 s, and 0.001 Hz corresponds to 1000 s. For extremely low frequencies the corresponding time could be years (eg 10 nHz = 3 years). The greater the frequency interval that you integrate over, the larger the resultant noise. Keep in mind, however, that extremely low-frequency noise measurements must be made over a long period of time. These phenomena will be discussed in greater detail in a later article. For now, please note that 0.1 Hz is often used for the lower cutoff frequency of the 1/f calculation.

Now we have both the broadband and 1/f noise magnitude. We must add these noise sources using the formula for uncorrelated noise sources given in Part I (see equation below and Equation 1.8 in Part I of this TechNote series).

$$e_{n_v} = \sqrt{e_{nf}^2 + e_{nBB}^2}$$

- e_{n_v} - total input referred voltage noise
- e_{nf} - total noise from 1/f
- e_{nBB} - total noise from broadband

Eq 2.6

A common concern that engineers have when considering this analysis technique is that they feel that the 1/f noise and broadband noise should be integrated in two separate regions. In other words, they believe that adding noise in this region will create an error because the 1/f noise will add with the broadband noise outside of the 1/f-region. The truth is that the 1/f-region extends across all frequencies as does the broadband-region. You must keep in mind that the noise spectrum is shown on a log chart and, so, the 1/f-region has little impact after it drops below the broadband curve. The only region where the combination of the two curves is obvious is near where they combine (often called the 1/f-corner frequency). In this region, you can see that the two sections combine as is described by our mathematical model. Fig. 2.12 illustrates how the two regions actually overlap as well as giving some relative magnitudes.

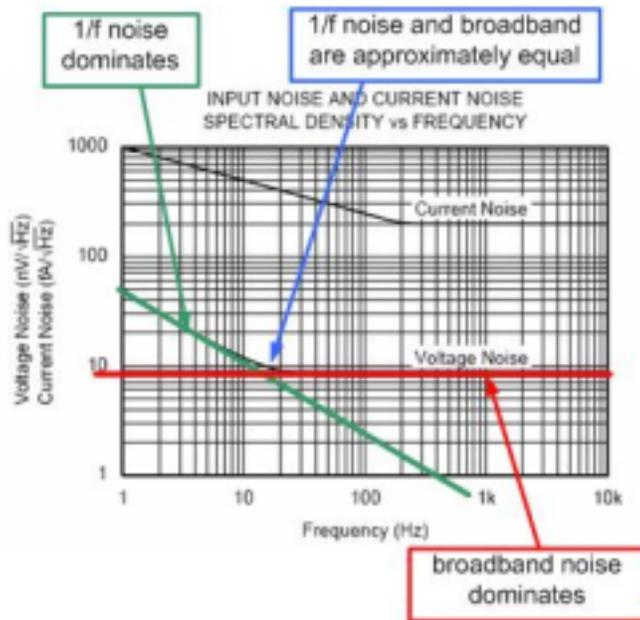


Fig. 2.12: 1/f Noise Region and Broadband Noise Regions Overlap

At this point we have developed all the equations necessary for converting a noise spectral density curve to a noise source. Note that the equations were derived for voltage noise, but the same technique works for current. In the next part of this article series, we will address the noise analysis of op amp circuits using these equations.

Summary And Preview

This part of the noise series introduced the op amp noise model and the noise spectral density curve. Also, some fundamental noise equations were introduced. Part III of this series will give examples of noise calculations using real world circuits.

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About The Author

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Appendix 2.1:

Derivation the “Brickwall” correction factor for a first order filter.

$$e_{\text{rms}}^2 = \int_{f_1}^{f_2} e_n^2 \cdot (|G|)^2 df$$

where

e_{rms} – total rms noise from f_1 to f_2 in Vrms

e_n – magnitude of noise spectral density at f_1 in $V/\sqrt{\text{Hz}}$

G – gain function for a single pole filter

$$G = \frac{1}{1 + \frac{j\omega}{\omega_p}} \quad |G| = \frac{1}{\sqrt{1^2 + \frac{f^2}{f_p^2}}} \quad (|G|)^2 = \frac{1}{1 + \frac{f^2}{f_p^2}}$$

$$e_{\text{rms}}^2 = \int_{f_1}^{f_2} e_n^2 \cdot \frac{1}{1 + \frac{f^2}{f_p^2}} df = \int_{f_1}^{f_2} e_n^2 \cdot \frac{f_p^2}{f_p^2 + f^2} df$$

$$e_{\text{rms}}^2 = e_n^2 \cdot f_p \cdot \text{atan}\left(\frac{f_2}{f_p}\right) - e_n^2 \cdot f_p \cdot \text{atan}\left(\frac{f_1}{f_p}\right)$$

Let $f_1 = 0$, $f_2 = \infty$

$$e_{\text{rms}}^2 = e_n^2 \cdot f_p \cdot \text{atan}(\infty) - e_n^2 \cdot f_p \cdot \text{atan}\left(\frac{f_1}{f_p}\right) = e_n^2 \cdot f_p \cdot \frac{\pi}{2}$$

$$e_{\text{rms}}^2 = e_n^2 \cdot f_p \cdot \frac{\pi}{2}$$

$$e_{\text{rms}} = \sqrt{e_n^2 \cdot f_p \cdot \frac{\pi}{2}}$$

Note that $\pi/2$ is $K_n = 1.57$ from Table 1.

Appendix 2.2:

Derivation the "Brickwall" correction factor for a first order filter.

$$e_n = \frac{e_{\text{normal}}}{f^{0.5}} \quad e_n^2 = \frac{e_{\text{normal}}^2}{(f^{0.5})^2} = \frac{e_{\text{normal}}^2}{f}$$

Units for

$$e_n = \text{V}/\sqrt{\text{Hz}}$$

$$e_{\text{normal}} = \text{V}$$

$$f = \text{Hz}$$

$$e_{\text{rms}}^2 = \int_a^b \frac{e_{\text{normal}}^2}{f} df = e_{\text{normal}}^2 \cdot \ln(f) \Big|_a^b$$

erms units of V rms

$$e_{\text{rms}}^2 = e_{\text{normal}}^2 \cdot \ln(b) - e_{\text{normal}}^2 \cdot \ln(a) = e_{\text{normal}}^2 \cdot \ln\left(\frac{b}{a}\right)$$

$$e_{\text{rms}}^2 = e_{\text{normal}}^2 \cdot \ln\left(\frac{b}{a}\right)$$

$$e_{\text{rms}} = e_{\text{normal}} \cdot \sqrt{\ln\left(\frac{b}{a}\right)}$$

$\sqrt{\ln\left(\frac{b}{a}\right)}$ has no units

e_{normal} has units of V